SENSITIVITY ANALYSIS OF LOSSY COUPLED TRANSMISSON LINES

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Abstract

An analysis method, based on the numerical inversion of the Laplace transform, is described for the evaluation of the time domain sensitivity of networks which include lossy coupled transmission lines. The sensitivity can be calculated with respect to network components and parameters of the transmission lines. Examples and comparisons with sensitivity determined by perturbation are presented.

I. Introduction

Analysis and design of interconnections in high-speed VLSI chips and printed circuit boards are gaining importance because of the rapid increase in operating frequencies and decrease in feature sizes. Improperly designed interconnects can result in increased signal delay, ringing, inadvertent and false switching [1]-[5]. This phenomena can be observed at the chip as well as the system level and the interconnected blocks could be analog, digital or mixed. With higher signal speeds, electrical length of interconnects can become significant fraction of a wavelength. Consequently, the conventional lumped impedance interconnect model is not adequate in this case. Instead, a distributed transmission line model should be used.

Recently, a new method [6] for the analysis of lossy coupled transmission lines with arbitrary linear terminal and interconnecting networks has been proposed. This method is based on a unified formulation technique for the equations describing the transmission line system and the equations describing the terminal and interconnecting networks. The time domain response is obtained using numerical inversion of Laplace transform (NILT) [7]-[8]. This method is more reliable and efficient than FFT-based methods [9].

Analysis or simulation tools allow the circuit designer to determine the response of a given network. Human expertise is still required to study the effect of variance of network parameters on system performance. Critical components which can affect rise/fall times, overshoot/undershoot, crosstalk, delay, etc. have to be identified through repeated circuit simulations. An important step is to use optimization techniques to speed up the design process. Transmission line effects such as crosstalk, delay and reflection can be minimized at vital connection ports. Application of powerful gradient based optimizers depends on the knowledge of sensitivities of the output responses. The direct perturbation approach of sensitivity analysis reduces the efficiency of optimization and can be very expensive if the number of optimization variables is large.

In this paper, a new method to evaluate the sensitivity of networks containing lossy coupled transmission lines is presented. The proposed approach is based on the numerical inversion of Laplace transform described in [6] and is more efficient and reliable compared to other methods based on FFT or the brute force perturbation technique. The developed tool can be used to identify critical network components or provide gradients to optimization routines.

II. Formulation of the Network Equations

Consider a linear network π which contains linear lumped components and arbitrary linear subnetworks. The linear lumped components can be described by equations in either the time or frequency domain. The arbitrary linear subnetworks may contain frequency dependent or distributed elements that are best described in the frequency domain. Without loss of generality the modified nodal admittance (MNA) matrix equation [10] of the network π can be written as

$$\mathbf{C}_\pi \frac{d\mathbf{v}_\pi(t)}{dt} + \mathbf{G}_\pi \mathbf{v}_\pi(t) + \sum_{k=1}^{N_s} \mathbf{D}_k \mathbf{i}_k(t) - \mathbf{e}_\pi(t) = 0 \qquad (1)$$

where

$\mathbf{C}_\pi(t) \in \Re^{N_\pi \times N_\pi}, \mathbf{G}_\pi(t) \in \Re^{N_\pi \times N_\pi}$ are constant matrices with entries determined by the lumped linear components,

$\mathbf{v}_\pi(t) \in \Re^{N_\pi}$ is the vector of node voltage waveforms appended by independent voltage source current and inductor current waveforms,

$\mathbf{D}_k = [d_{i,j}] \in \{0,1\}, i \in \{1,2,\ldots,N_\pi\}, j \in \{1,2,\ldots,N_k\}$ with a maximum of one nonzero in each row or column is a selector matrix that maps $\mathbf{i}_k(t) \in \Re^{N_k}$, the vector of currents entering the linear subnetwork k, into the node space $\Re^{N_\pi}$ of the network π,

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N_s is the number of linear subnetworks and

$\mathbf{e}_\pi(t) \in \Re^{N_\pi}$ is the vector of source waveforms.

The frequency domain representation can be obtained by taking the Laplace transform of (1):

$$[s\mathbf{C}_\pi + \mathbf{G}_\pi]\mathbf{V}_\pi(s) + \sum_{k=1}^{N_s} \mathbf{D}_k \mathbf{I}_k(s) = \mathbf{E}_\pi(s) + \mathbf{C}_\pi \mathbf{v}_\pi(0) \quad (2)$$

Let the frequency domain equations of the linear subnetwork k to be in the form

$$\mathbf{A}_k \mathbf{V}_k(s) = \mathbf{I}_k(s) \quad (3)$$

where $\mathbf{V}_k(s)$ and $\mathbf{I}_k(s)$ represent the frequency domain terminal voltages and currents of the subnetwork k and $\mathbf{A}_k$ represents the MNA matrix of the subnetwork.

Combining (2) and (3) produces

$$\mathbf{Y}_\pi \mathbf{V}_\pi = \mathbf{E}_\pi + \mathbf{C}_\pi \mathbf{v}_\pi(0) \quad (4)$$

where

$$\mathbf{Y}_\pi = s\mathbf{C}_\pi + \mathbf{G}_\pi + \sum_{k=1}^{N_s} \mathbf{D}_k \mathbf{A}_k \mathbf{D}_k^t$$

In the case where the subnetwork k consists of a transmission line with N conductors, $\mathbf{A}_k$ is given by

$$\mathbf{A}_k = \begin{bmatrix} \mathbf{S}_i\mathbf{E}_1\mathbf{S}_v^{-1} & \mathbf{S}_i\mathbf{E}_2\mathbf{S}_v^{-1} \\ \mathbf{S}_i\mathbf{E}_2\mathbf{S}_v^{-1} & \mathbf{S}_i\mathbf{E}_1\mathbf{S}_v^{-1} \end{bmatrix} \quad (5)$$

$\mathbf{E}_1$ and $\mathbf{E}_2$ are diagonal matrices

$$\mathbf{E}_1 = \mathrm{diag}\left\{\frac{e^{-2\gamma_i D} + 1}{1 - e^{-2\gamma_i D}}, i = 1 \ldots N\right\} \quad (6)$$

$$\mathbf{E}_2 = \mathrm{diag}\left\{\frac{2}{e^{-\gamma_i D} - e^{\gamma_i D}}, i = 1 \ldots N\right\} \quad (7)$$

D is the length of the line,

γ_i^2 is an eigenvalue of the matrix $\mathbf{Z}_L\mathbf{Y}_L$ with an associated eigenvector $\mathbf{x}_i$, where

$$\mathbf{Z}_L = \mathbf{R} + s\mathbf{L} \quad (8)$$

$$\mathbf{Y}_L = \mathbf{G} + s\mathbf{C} \quad (9)$$

$\mathbf{R}$, $\mathbf{L}$, $\mathbf{C}$, and $\mathbf{G}$ represent the line parameters per unit length,

$\mathbf{S}_v$ is a matrix with the eigenvectors $\mathbf{x}_i$ in the columns,

$\mathbf{S}_i = \mathbf{Z}_L^{-1}\mathbf{S}_v\mathbf{\Gamma}$ and

$\mathbf{\Gamma}$ is a diagonal matrix with $\Gamma_{i,i} = \gamma_i$.

III. Sensitivity Analysis

Consider the system of linear equations described by (4). Define the output of the system to be

$$\phi = \mathbf{d}^t \mathbf{V}_\pi \quad (10)$$

where $\mathbf{d}$ is a constant vector.

Differentiating (4) and substituting (10) results in

$$\frac{\partial \phi}{\partial \lambda} = (\mathbf{V}_\pi^a)^t \left[\frac{\partial \mathbf{Y}_\pi}{\partial \lambda}\mathbf{V}_\pi - \frac{\partial \mathbf{E}_\pi}{\partial \lambda} - \frac{\partial \mathbf{C}_\pi}{\partial \lambda}\mathbf{v}_\pi(0)\right] \quad (11)$$

where λ is a parameter of the network and $\mathbf{Y}_\pi^t \mathbf{V}_\pi^a = -\mathbf{d}$.

If λ is a parameter of the transmission line k (11) is reduced to

$$\frac{\partial \phi}{\partial \lambda} = (\mathbf{V}_\pi^a)^t \left[\mathbf{D}_k \frac{\partial \mathbf{A}_k}{\partial \lambda}\mathbf{D}_k^t\right] \mathbf{V}_\pi \quad (12)$$

From (5) it can be shown that

$$\begin{aligned} \frac{\partial \mathbf{A}_k}{\partial \lambda}\begin{bmatrix} \mathbf{S}_v & 0 \\ 0 & \mathbf{S}_v \end{bmatrix} &= \begin{bmatrix} \frac{\partial \mathbf{S}_i}{\partial \lambda} & 0 \\ 0 & \frac{\partial \mathbf{S}_i}{\partial \lambda} \end{bmatrix}\begin{bmatrix} \mathbf{E}_1 & \mathbf{E}_2 \\ \mathbf{E}_2 & \mathbf{E}_1 \end{bmatrix} \\ &+ \begin{bmatrix} \mathbf{S}_i & 0 \\ 0 & \mathbf{S}_i \end{bmatrix}\begin{bmatrix} \frac{\partial \mathbf{E}_1}{\partial \lambda} & \frac{\partial \mathbf{E}_2}{\partial \lambda} \\ \frac{\partial \mathbf{E}_2}{\partial \lambda} & \frac{\partial \mathbf{E}_1}{\partial \lambda} \end{bmatrix} \\ &- \mathbf{A}_k \begin{bmatrix} \frac{\partial \mathbf{S}_v}{\partial \lambda} & 0 \\ 0 & \frac{\partial \mathbf{S}_v}{\partial \lambda} \end{bmatrix} \end{aligned} \quad (13)$$

where

$$\mathbf{Z}_L \frac{\partial \mathbf{S}_i}{\partial \lambda} = \frac{\partial \mathbf{S}_v}{\partial \lambda}\mathbf{\Gamma} + \mathbf{S}_v \frac{\partial \mathbf{\Gamma}}{\partial \lambda} - \frac{\partial \mathbf{Z}_L}{\partial \lambda}\mathbf{S}_i$$

$\partial \mathbf{A}_k / \partial \lambda$ is dependent on the sensitivity of the eigenvalues γ_i^2 and eigenvectors $\mathbf{x}_i$ of the matrix $\mathbf{Z}_L\mathbf{Y}_L$. It can be shown that the sensitivity of γ_i^2 and $\mathbf{x}_i$ is given by

$$\begin{bmatrix} \gamma_i^2 \mathbf{U} - \mathbf{Z}_L\mathbf{Y}_L & \mathbf{x}_i \\ \mathbf{x}_i^t & 0 \end{bmatrix}\begin{bmatrix} \frac{\partial \mathbf{x}_i}{\partial \lambda} \\ \frac{\partial \gamma_i^2}{\partial \lambda} \end{bmatrix} = \begin{bmatrix} \frac{\partial(\mathbf{Z}_L\mathbf{Y}_L)}{\partial \lambda}\mathbf{x}_i \\ 0 \end{bmatrix} \quad (14)$$

Once all the separate partial derivatives have been calculated and combined, the resultant $\partial \mathbf{A}_k / \partial \lambda$ can be inserted into (12) to produce the frequency domain sensitivity $\partial \phi / \partial \lambda$. It should be noted that λ could be an electrical parameter of the transmission line or a physical parameter.

Given a frequency domain response for the circuit sensitivity $\partial \phi(s) / \partial \lambda$, the time domain response is

$$\frac{\partial \hat{\phi}(t)}{\partial \lambda} = -(1/t)\sum_{i=1}^{M'} \mathrm{Re}\left[K_i' \frac{\partial \phi(z_i/t)}{\partial \lambda}\right] \quad (15)$$

where M', K_i' and z_i are related to the Padé approximation of the exponential function e^z.

IV. Numerical Example

The circuit in Fig. 1 contains three lossy coupled transmission lines. The lengths of transmission lines 1, 2 and 3 are 0.05m, 0.04m and 0.03m respectively. The transmission line parameters for the three lines are identical:

$$\mathbf{L} = \begin{bmatrix} 494.6 & 63.3 \\ 63.3 & 494.6 \end{bmatrix} \mathrm{nH/m}, \quad \mathbf{C} = \begin{bmatrix} 62.8 & -4.9 \\ -4.9 & 62.8 \end{bmatrix} \mathrm{pF/m},$$

$$\mathbf{R} = \begin{bmatrix} 75 & 15 \\ 15 & 75 \end{bmatrix} \Omega/\mathrm{m}, \quad \mathbf{G} = \begin{bmatrix} 0.1 & -0.01 \\ -0.01 & 0.1 \end{bmatrix} \mathrm{S/m}.$$

The applied voltage for this example is shown in Fig. 2. The time domain sensitivities of the output voltage V_{out} obtained using the proposed method are shown in Fig. 3–Fig. 5 which demonstrate excellent agreement with the results obtained by perturbation.

V. Conclusion

An analysis method, based on the NILT algorithm, has been described for the evaluation of the time domain sensitivity of networks which contain lossy coupled transmission lines. The sensitivity can be calculated with respect to network components and electrical and physical parameters of the transmission lines. The efficiency of the proposed method makes it useful for determining critical network components and for generating sensitivities of output responses used in gradient based optimizers.

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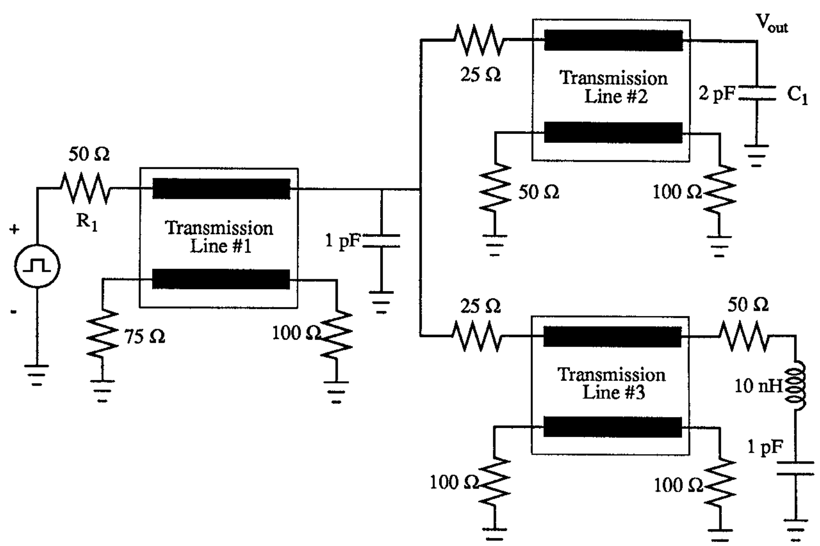

Figure 1: Interconnect model with lossy coupled transmission lines.

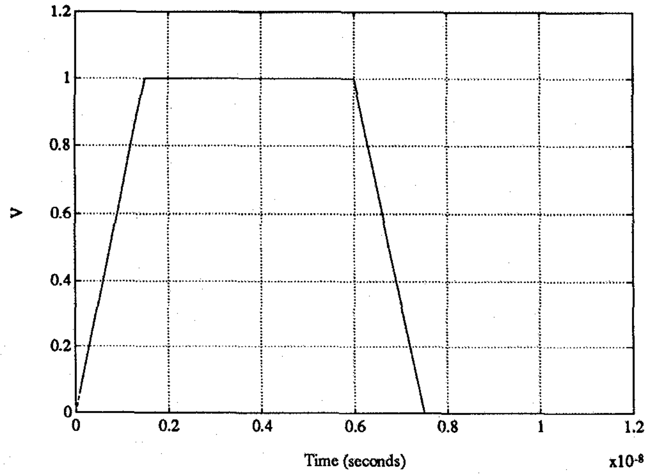

Figure 2: Applied input voltage waveform used for circuit in Fig. 1.

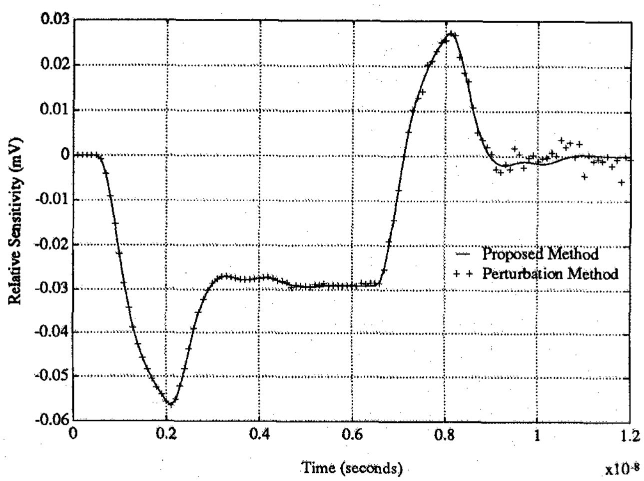

Figure 4: Sensitivity of voltage V_{out} with respect to parameter $R_{1,1}$ of transmission line #2 in Fig. 1.

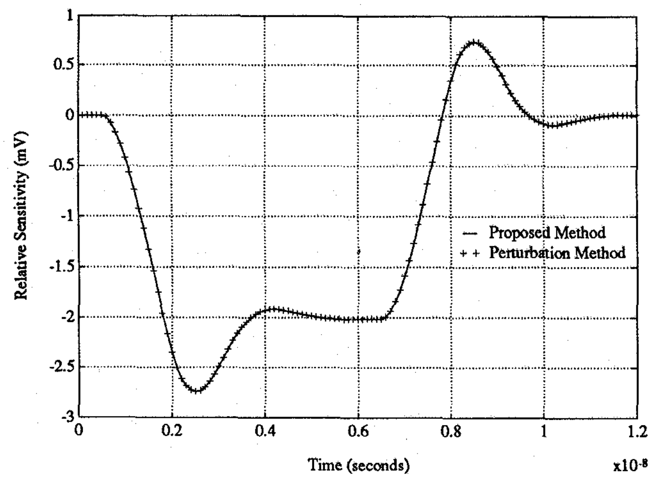

Figure 3: Sensitivity of voltage V_{out} with respect to R_1 of Fig. 1.

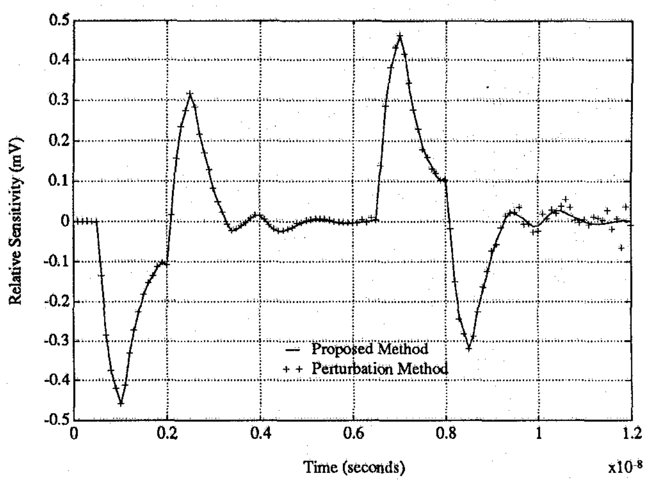

Figure 5: Sensitivity of voltage V_{out} with respect to parameter $L_{1,1}$ of transmission line #2 in Fig. 1.